

1.1 Radical Expressions: Rationalizing Denominators

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- To rewrite a radical expression with a one-term radical in the denominator, multiply the numerator and denominator by the one-term denominator.

$$\frac{\sqrt{a}-\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} \\ = \frac{\sqrt{ab}}{b}$$

- When the denominator of a radical expression is a two-term expression, rationalize the denominator by multiplying the numerator and denominator by the conjugate and then simplify.

$$\frac{1}{\sqrt{a}-\sqrt{b}} = \frac{1}{\sqrt{a}-\sqrt{b}} \times \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}} \\ = \frac{\sqrt{a}+\sqrt{b}}{a-b}$$

- $\sqrt{a}+\sqrt{b}$ is the conjugate of $\sqrt{a}-\sqrt{b}$, and vice versa
so,

$$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b \\ (1+\sqrt{2})(1-\sqrt{2}) = 1-2 = -1$$

- conjugate guarantees the middle term cancels

- $\sqrt{a} \times \sqrt{a} = a \quad a \geq 0$

ex. Rationalize denominator

$$\frac{\sqrt{3}}{\sqrt{2}+\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{2}+\sqrt{5}} \times \frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}} = \frac{\sqrt{6}-\sqrt{15}}{2-5} = \frac{\sqrt{6}-\sqrt{15}}{-3}$$

Rationalize the numerator

$$\frac{\sqrt{3}}{\sqrt{2}+\sqrt{5}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3}{\sqrt{6}+\sqrt{15}}$$

ex. $\frac{\sqrt{a+h}-\sqrt{a}}{h} \times \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}} = \frac{a+h-a}{h(\sqrt{a+h}+\sqrt{a})} = \frac{1}{\sqrt{a+h}+\sqrt{a}}$

* The original function was $f(x) = \sqrt{x}$

6f) ex. $\frac{\sqrt{18} + \sqrt{12}}{\sqrt{18} - \sqrt{12}} = \frac{\sqrt{9}\sqrt{2} + \sqrt{4}\sqrt{3}}{\sqrt{9}\sqrt{2} - \sqrt{4}\sqrt{3}} = \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} \times \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}}$

Rationalize denominator

$$= \frac{18 + 6\sqrt{3}\sqrt{2} + 6\sqrt{2}\sqrt{3} + 12}{18 - 6\sqrt{3}\sqrt{2} + 6\sqrt{3}\sqrt{2} - 12}$$

$$= \frac{18 + 2(\sqrt{3}\sqrt{2} \times 6) + 12}{6}$$

$$= \frac{30 + 12\sqrt{3}\sqrt{2}}{6}$$

$$= 5 + 2\sqrt{6}$$

• When you simplify a radical expression such as $\frac{\sqrt{3}}{\sqrt{2}}$, multiply the numerator and denominator by the radical only.

$$\frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{6}}{2 \cdot 2} = \frac{\sqrt{6}}{4}$$

$$\bullet (\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{a}\sqrt{b}$$

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1. a) $2\sqrt{3} + 4$
- b) $\sqrt{3} - \sqrt{2}$
- c) $-2\sqrt{3} + \sqrt{2}$
- d) $3\sqrt{3} - \sqrt{2}$
- e) $\sqrt{2} + \sqrt{5}$
- f) $-\sqrt{5} - 2\sqrt{2}$

2. a) $\frac{\sqrt{3} + \sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{10}}{2}$
- b) $\frac{2\sqrt{3} - 3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{6} - 6}{2} = \sqrt{6} - 3, -3 + \sqrt{6}$
- c) $\frac{4\sqrt{3} + 3\sqrt{2}}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{8\sqrt{9} + 6\sqrt{6}}{12} = \frac{24 + 6\sqrt{6}}{12} = 2 + \frac{1}{2}\sqrt{6}$
- d) $\frac{5\sqrt{5} - \sqrt{2}}{2\sqrt{2}} \times \frac{2\sqrt{2}}{2\sqrt{2}} = \frac{6\sqrt{10} - 2\sqrt{4}}{4\sqrt{4}} = \frac{6\sqrt{10} - 4}{8} = \frac{3}{4}\sqrt{10} - \frac{1}{2}$

$$3. a) \frac{5}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3\sqrt{5}+3\sqrt{2}}{5-2}$$

$$b) \frac{2\sqrt{5}}{2\sqrt{5}+3\sqrt{2}} \times \frac{2\sqrt{5}-3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{(4\sqrt{5})-6\sqrt{10}}{(4\sqrt{5})-9\sqrt{4}} = \frac{20-6\sqrt{10}}{20-18} = \frac{20-6\sqrt{10}}{2}$$

$$= 10-3\sqrt{10}$$

$$c) \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{3-\sqrt{6}+\sqrt{6}+2}{3-2} = 5-2\sqrt{6}$$

$$d) \frac{2\sqrt{5}-8}{2\sqrt{5}+3} = \frac{2\sqrt{5}-3}{2\sqrt{5}-3} = \frac{(4\sqrt{5})-16\sqrt{5}-6\sqrt{5}+24}{20+6\sqrt{5}-6\sqrt{5}-9} = \frac{44-22\sqrt{5}}{11}$$

$$= 4-2\sqrt{5}$$

$$e) \frac{2\sqrt{3}-\sqrt{2}}{\sqrt{2}+\sqrt{3}} \times \frac{5\sqrt{2}-\sqrt{3}}{5\sqrt{3}-\sqrt{3}} = \frac{10\sqrt{6}-10+6+\sqrt{6}}{25\sqrt{6}-3} = \frac{11\sqrt{6}-16}{25\sqrt{6}-3}$$

$$f) \frac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}} \times \frac{3\sqrt{3}-2\sqrt{2}}{3\sqrt{3}-2\sqrt{2}} = \frac{18-6\sqrt{6}-6\sqrt{6}+8}{18-6\sqrt{6}+6\sqrt{6}-8} = \frac{26-12\sqrt{6}}{10}$$

$$4. a) \frac{\sqrt{5}-1}{4} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{5-1}{4(\sqrt{5}+1)} = \frac{4}{4(\sqrt{5}+1)}$$

$$b) \frac{2-3\sqrt{2}}{2} \times \frac{2+3\sqrt{2}}{\sqrt{2}+3\sqrt{2}} = \frac{4-6\sqrt{2}+6\sqrt{2}-18}{4+6\sqrt{2}} = \frac{-14}{4+6\sqrt{2}}$$

$$c) \frac{\sqrt{5}+2}{2\sqrt{5}-1} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} = \frac{5-4}{20-\sqrt{5}-4\sqrt{5}+2} = \frac{1}{22-5\sqrt{5}}$$

$$5. a) \frac{8\sqrt{2}}{\sqrt{20}-\sqrt{18}} \times \frac{\sqrt{20}+\sqrt{18}}{\sqrt{20}+\sqrt{18}} = \frac{8\sqrt{40}+8\sqrt{36}}{20-18} = \frac{8\sqrt{4}\sqrt{10}+8\sqrt{9}\sqrt{4}}{20-18}$$

$$= \frac{16\sqrt{10}+48}{2} = 8\sqrt{10}+24$$

$$b) 8\sqrt{10}+24$$

$$c) \text{ b/c } \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$$

$$\text{ and } \sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$$

$$6. a) \frac{\sqrt{8}}{\sqrt{2}-\sqrt{8}} \times \frac{\sqrt{2}+\sqrt{8}}{\sqrt{2}+\sqrt{8}} = \frac{4\sqrt{2}+8}{12-8} = \sqrt{2}+2$$

$$b) \frac{9\sqrt{2}+2\sqrt{3}}{25}$$

$$c) 2\sqrt{2}+\sqrt{6}$$

$$d) \frac{12+5\sqrt{6}}{2}$$

$$e) \frac{12\sqrt{15}+15\sqrt{10}}{2}$$

$$f) 5+2\sqrt{6}$$