10b.

$$\frac{50\left[(10-h)^2-10^2\right]}{h}$$

$$\frac{50(10-h-10)(10-h+10)}{h}$$

$$\frac{50(-\cancel{h})(20-h)}{\cancel{h}} = -50(20-h)$$

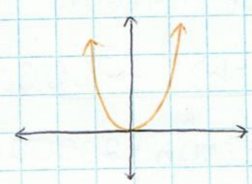
$$= -1000+50h$$

$$= -1000$$

## 1.4 Limit of a Function

- $\lim_{x \to a} f(x)$ means limit of $f(x)$ a $x$ approaches $a$
- what is the m-value approaching when $x$ is approaching a specific ~~x~~- value (a)

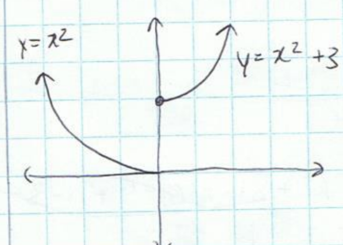
$\lim_{x \to 0} (fx) = 0$



$\rightarrow \lim_{x \to 2} f(x) = 4$

for $f(x) = x^2$

For Piecewise Functions




$f(0) = 0$

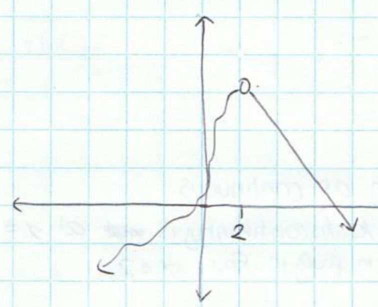
$\lim_{x \to 0} f(x) =$ doesn't exist

because

$\lim_{\substack{x \to 0^+ \\ x \to 0^-}} f(x) = \begin{cases} \nearrow \lim_{x \to 0^+} f(x) = 3 \\ \searrow \lim_{x \to 0^-} f(x) = 0 \end{cases}$

- we can't have 2 random values, so

$\lim_{x \to 0} f(x) =$ doesn't exist




- limit exists at 2 still because another would be the same

- At a hole the limit does exist b/c the limit means 'approaches' but not equal to

- one sided limit

- $\lim_{x \to a^+} f(x)$ - means limit of $f(x)$ as $x$ approaches 'a' from the right side

- $\lim_{x \to a^-} f(x)$ - means limit of $f(x)$ as $x$ approaches 'a' from the left side

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ex.2 a) $\lim_{x \to 1} \frac{x^2-1}{x-1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$\rightarrow$ there is a hole so $y = \lim_{x \to 1} (x+)$

$x = 1+1$

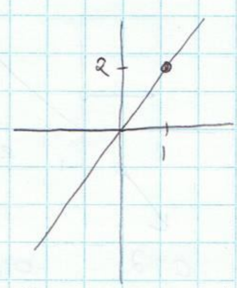
$= 2$

from right side

| $x$ | $f(x)$ |
|---|---|
| 1.1 | 2.1 |
| 1.03 | 2.05 |
| 1.04 | 2.04 |
| 1.04 | 2.03 |
| 1.02 | 2.02 |
| 1.01 | 2.01 |

from left side

| $x$ | $f(x)$ |
|---|---|
| 0.9 | 1.9 |
| 0.99 | 1.99 |




- as $x$ approaches 1, $y$ approaches 2

b) if $\lim_{x \to 1}(2x+3)$

then $\lim_{x \to 1}(2x+3) = 5$

- you can use reciprocal function as continuous

$\lim_{x \to 2} \frac{1}{x^2-1}$ - no problem b/c discontinuity is ~~not~~ at $x = \pm 1$
- ~~but~~ you can plug in for $x=2$

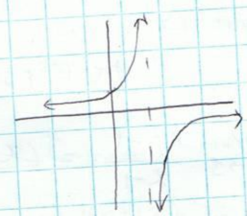
$\lim_{x \to 2} \frac{1}{3}$

Discontinuity is from → hole - circular - exist

→ jump - discontinuous

→ vertical asymptote - denominator - doesn't exist

c) $\lim_{x \to 1} \frac{1}{x-1}$



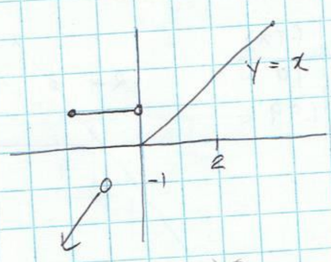
VA → 1
so limit doesn't exist

Show

$\lim_{x \to 1^+} \frac{1}{x-1} = +\infty$ $\qquad$ $\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$

$\lim_{x \to 1} \frac{1}{x-1}$ = does not exist

ex. 3




a) $\lim_{x \to 2} f(x)$ $\quad$ b) $\lim_{x \to 0^+} f(x)$ $\quad$ c) $\lim_{x \to 0^-} f(x)$

d) $\lim_{x \to -3} f(x)$ $\quad$ e) $\lim_{x \to -1} f(x)$ $\quad$ f) $\lim_{x \to 0} f(x)$

a) 2 $\quad$ b) 0 $\quad$ c) 1 $\quad$ d) -3

e) does not exist because limit from left and right is different

f) does not exist

$$f(x) = \begin{cases} x & , \ x \geq 0 \\ -x & ; \ x < -1 \\ 1 & ; \ -1 \leq x < 0 \end{cases}$$

1) If limit from right side of ~~a~~ doesn't equal $\lim_{x \to a^-} f(x)$

then $\lim_{x \to a} f(x)$ = doesn't exist

$$\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$$

2) if $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$

then $\lim_{x \to a} f(x) = L$

- piecewise functions are those where values aren't same

$$f(x) = \begin{cases} x^2 - 1 & , \ x \geq 5 \\ x + 23 & ; \ x < 5 \end{cases}$$

$\lim_{x \to 5^+} f(x) = 24$ $\qquad$ $\lim_{x \to 5^-} f(x) = 28$

$x^2 - 1$ $\qquad$ $x + 23$

let $x = .727272$

$100x = 72.7272$

$100x - x = 72$

$99x = 72$

$x = \frac{72}{99} = 0.727272$

# 1.4 The Limit of a Function

## Limits and their existence

- We say that the number L is the limit of a function $y=f(x)$ as $x$ approaches the value 'a', written as $\lim_{x\to a} f(x) = L$, if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$.
- Otherwise, $\lim_{x\to a} f(x)$ does not exist.

- The limit must approach from left and right side.

- $\lim_{x\to a} f(x)$ may exist even if $f(a)$ is not defined.
- $\lim_{x\to a} f(x)$ can be equal to $f(a)$. In this case, the graph of $f(x)$ passes through the point $(a, f(a))$.
- If $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$, then L is the limit of $f(x)$ as $x$ approaches a, that is $\lim_{x\to a} f(x) = L$.

## Practise

1. a) let $x = 0.7272\ldots$

$$100x = 72.727\ldots$$

$$100x - x = 72$$

$$\frac{99x}{99} = \frac{72}{99} = 0.72\ldots$$

$$x \to \frac{72}{99}$$

b) let $x = 3.141592\ldots$

$$x \to \pi$$

2. Evaluate the function for the values of the independent variable that progressively get closer to the given value of the independent variable