2.1 The Derivative Function

Feb.18.2013

Pg.65

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- we need to use above formula without plugging in 'a'.

ex. $f(x) = x^2$

$m = 2x$ ← derivative

← slope value for all x-values

↓

written as

$f'(x) = 2x$

f prime at x

-derivative of $f(x)$

ex. Find $f'(x)$ given $f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$ ← you can also use difference of squares $a^2 - b^2 = (a-b)(a+b)$

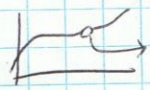
$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$f'(x) = 2x$

- The **derivative** of $f(x)$ at the point x is given by
  $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ provided the limit exists
- Use the long way when it says *use the definition of derivative"
  * first principle
  * difference of quotient"
- Not continuous implies not differentiable
  ↳ can't find slope.

ex. hole



→ no limit defined

- $f(x)$ is said to be differenciable at $x = a$ if $f(a)$ exists.

ex. $f(x) = \frac{1}{x-1}$ VA at $x = 1$

↳ differenciability – all values of domain except at $x=1$

y' and $f'(x)$

↳ no. defined independent variable

↳ independent variable is always defined at x, or any variable

y' is the derivative of y

- **Leibniz Notation** – $\frac{dy}{dx}$ → derivative of y with respect to x

– y' won't show this

ex. $f'(q) \rightarrow \frac{dy}{dq}$

$$y = -q^2 + 10 + p$$

$\frac{dy}{dq} = -2q$ $\quad$ $\frac{dy}{dp} = 1$ ← no exact independent variable brings 2 derivatives

'd' stands for derivative

ex. $f(x) = \frac{1}{x}$ $\quad$ $f'(x) = \frac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{x}{x+h} - \frac{(x+h)}{x}}{h} = \lim_{h\to 0} \frac{\cancel{x} - \cancel{x} - \cancel{h}}{(x)(x+h)(\cancel{h})} = \frac{-1}{x^2}$$

- not differentiable for $x=0$
- infinite discontinuity
- VA

Find the eq^n of tangent at $x=2$

$x = 2$

$y = 1/2$

$m = -1/4$

$y = mx + b$

$\frac{1}{2} = \cancel{2}\left(\frac{-1}{\cancel{4}_2}\right) + b$

$\frac{1}{2} + \frac{1}{2} = b$

$b = 1$

$y = -\frac{1}{4}x + 1$

ex. 3 $f(x) = \sqrt{x} = x^{1/2}$ $f'(x) = \frac{1}{2}(x)^{-1/2}$

$$f'(x) = \frac{\sqrt{x+h} - \sqrt{x}}{h} \qquad = \frac{1}{2\sqrt{x}}$$

$$= \frac{\cancel{x}+h-\cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{2\sqrt{x}}$$

- At a cusp, a function is not differenciable
- At a discontinuity, a function is not differenciable
- At a vertical tangent, a function is not differenciable

ex. $f(x) = \frac{x^2+1}{x}$ $\quad \frac{x^2}{x} + \frac{1}{x}$ $\quad f'(x) = x + \frac{1}{x}$

$$= \frac{x^2}{x^2} - \frac{1}{x^2}$$

$$= \frac{x^2-1}{x^2}$$

$$\frac{x(x^2+h^2+2xh+1)}{\cancel{x+h}} - \frac{(x^2+1)(x+h)}{x}$$

$$\frac{x^3 + xh^2 + 2x^2h + x^2 - (x^3 + x + hx^2 + h)}{(x)(x+h)(h)}$$

$$\frac{\cancel{x^3} + xh^2 + x^2h + \cancel{x} - \cancel{x^3} - \cancel{x} - \cancel{hx^2} - h}{x(x+h)(h)}$$

$$\frac{\cancel{h}(xh + x^2 - 1)}{x(\cancel{h})(x+h)} \quad \frac{x^2-1}{x^2}$$

~~2.2~~

6. d) $\sqrt{3x+2}$ $\quad f'(x) = \frac{3\cancel{h}}{\cancel{h}(\sqrt{3(x+h)} + \sqrt{3x+2})} = \ = \frac{3}{2\sqrt{3x+2}}$

7. b) $y = \frac{x+1}{x-1}$ $\quad f'(x) = \frac{x+h+1\,(x-1) - (x+1)(x+h-1)}{(x+h)(x-1)(h)}$

$$= \frac{\cancel{x^2} \cancel{x} + \cancel{xh} - \cancel{1} - \cancel{x} \cancel{1} - \cancel{x^2} \cancel{xh} + \cancel{x} \cancel{x} - h + 1}{(x+h-1)(x-1)(\cancel{h})}$$

$$= \frac{-2}{(x+h-1)(x-1)}$$

$$= \frac{-2}{x^2-2x+1}$$

2.1 The Derivative Function

Feb. 22. 2014

- The ~~de~~ derivative of f at the number a is given by
$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ provided that this limit exists

- An alternative way of writing the derivative of f at the number ~~a~~ is $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

The Definition of the Derivative Function

- The derivative of $f(x)$ with respect to x is the function $f'(x)$, where $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, provided that this limit exists

- The normal to the graph of f at point P is the line that is perpendicular to the tangent at P.

- The derivative of a function f at a point $(a, f(a))$ is
$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ or $f'(a) = \lim_{h \to 0} \frac{f(x) - f(a)}{x - a}$
if the limit exists.

- A function is said to be differentiable at a if $f'(a)$ exists. A function is differentiable on an interval if it is differentiable at every number in the interval.

- The derivative function for any function $f(x)$ is given by $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, if the limit exists.