- To find the derivative at a point $x = a$, you can use $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

- The derivative $f(a)$ can be interpreted as either
  - the slope of the tangent at $(a, f(a))$, or
  - the instantaneous rate of change of $f(x)$ with respect to x when $x = a$

- Other notations for the derivative of the function $y = f(x)$ are $f'(x)$, y' and $\frac{dy}{dx}$

- The normal to the graph of a function at point P, is a line that is perpendicular to the tangent line that passes through point P.

2.2 Derivatives of Polynomial Functions

Feb. 19. 2013

$f(x) = x^2 \qquad f'(x) = 2x$

$f(x) = \frac{1}{x} \qquad f'(x) = \frac{-1}{x^2}$

$f(x) = \sqrt{x} \qquad f'(x) = \frac{1}{2\sqrt{x}}$

$f(x) = x^3 \qquad f(x) = 3x^2$

$f(x) = \frac{1}{x^{1/3}} \qquad x^{-1/3 - 1} \qquad \frac{1}{3} x^{-4/3} \qquad -\frac{1}{3\sqrt[3]{x^4}}$

- If $f(x) = x^n$
  $f'(x) = n x^{n-1}$

- If $f(x) = k \quad f'(x) = 0 \rightarrow$ derivative of a number is 0.

- $f(x) = x \qquad f'(x) = 1$

- $f(x) = ax \qquad f'(x) = a$

Ex. 1 a) $y = 3x^2 - 5x + 10$ $6x - 5$

b) $y = x^{10} + \sqrt{x}$ $2\sqrt{x}\,\frac{10x^9}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} = \frac{20\sqrt{x^{19}}+1}{2\sqrt{x}}$

c) $y = \frac{4}{x^2}$

d) $y = 10x^{1/2}$

e) $y = \frac{x^{10} + x^5}{x^2}$

c) $y = \frac{4}{x^2} = 4x^{-2} = -8x^{-3}$

d) $y = 5x^{-1/2} = \frac{5}{\sqrt{x}}$

e) $\frac{x^{10}}{x^2} + \frac{x^5}{x^2}$

$x^8 + x^3$

$8x^7 + 3x^2$

2.2 The Derivatives of Polynomial Functions

Rule	Function Notation	Leibniz notation
① Constant Function Rule	If $f(x)=k$, where k is a constant, $f'(x)=0$	$\frac{d}{dx}(k)=0$
② Power Rule	If $f(x)=x^n$, $n \in \mathbb{R}$ then $f'(x)=nx^{n-1}$	$\frac{d}{dx}(x^n)=nx^{n-1}$
③ Constant Multiple Rule	If $f(x)=kg(x)$, $f'(x)=kg'(x)$	$\frac{d}{dx}(ky)=k\frac{dy}{dx}$
④ Sum Rule	If $f(x)=p(x)+q(x)$, $f'(x)=p'(x)+q'(x)$	$\frac{d}{dx}f(x)=\frac{d}{dx}p(x)+\frac{d}{dx}q($
⑤ Difference Rule	If $f(x)=p(x)-q(x)$ $f'(x)=p'(x)-q'(x)$	$\frac{d}{dx}f(x)=\frac{d}{dx}p(x)-\frac{d}{dx}(q(x)$

Proofs - using definition

① $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{k-k}{h}=\lim_{h\to0}0=0$

* $f(x)=k$ for all x-values

② $n \in I\ (+)$

$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$, where $f(x)=x^n$

$=\lim_{h\to0}\frac{(x+h)^n-x^n}{h}=\lim_{h\to0}\frac{\cancel{(x+h-x)}\left[(x+h)^{n-1}+(x+h)^{n-2}x\ldots(x^{n-1})\right]}{\cancel{h}}$

$=\lim_{h\to0}\left[(x+h)^{n-1}+(x+h)^{n-2}\ldots+(x+h)^{n-2}+x^{n-1}\right]$

$=x^{n-1}+x^{n-2}x\ \ldots\ x(x^{n-2})+x^{n-1}$

$=x^{n-1}+x^{n-1}\ldots+x^{n-1}+x^{n-1}$

$=nx^{n-1}$

③ $f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{k(g(x+h))-k(g(x))}{h}$

$=k\times\lim_{h\to0}\left[\frac{g(x+h)-g(x)}{h}\right]=kg'(x)$

4) 5) $F'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{p(x+h) + q(x+h) - p(x) - q(x)}{h}$

$= \lim_{h \to 0} \frac{p(x+h) - p(x)}{h} + \frac{q(x+h) - q(x)}{h}$

$= \lim_{h \to 0} \left(\frac{p(x+h) - p(x)}{h}\right) + \lim_{h \to 0} \left(\frac{p(x+h) - p(x)}{h}\right)$

$= p'(x) + q'(x)$

- To determine the derivative of a simple rational function, such as $f(x) = \frac{4}{x^6}$, express the function as a power, then use the power rule.

  If $f(x) = 4x^{-6}$, then $f'(x) = 4(-6)x^{(-6-1)} = -24x^{-7}$

- If you have a radical function such as $g(x) = \sqrt[3]{x^5}$, rewrite the function as $g(x) = x^{5/3}$, then use the power rule.

  If $g(x) = x^{5/3}$, then $g'(x) = \frac{5}{3} x x^{2/3} = \frac{5}{3} x\sqrt[3]{x^2}$

2.3 The Product Rule

1) $y = (2x)^2 \quad 4x^2 \rightarrow 8x$

2) $y = (2x+1)^2 \quad \cancel{4}x^2 + 2x + 2x + 1 = 4x^2 + 4x + 1 \rightarrow y' = 8x + 4$

3) $y = 3\sqrt{x} + \sqrt{x} + \frac{1}{x} + \frac{1}{x^2} \rightarrow x^{-2} \quad -2x^{-3}$

$x^{1/3} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

$x^{1/2} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\frac{-1}{x^2}$

$-\frac{2}{x^3}$

$y' = \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{2\sqrt{x}} - \frac{1}{x^2} - \frac{2}{x^3}$

$(2x+1)^2$

→ multiply the derivative by the slope of the inside.