① ⑤ $F'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{p(x+h) + q(x+h) - p(x) - q(x)}{h}$

$= \lim_{h \to 0} \frac{p(x+h) - p(x)}{h} + \frac{q(x+h) - q(x)}{h}$

$= \lim_{h \to 0} \left(\frac{p(x+h) - p(x)}{h}\right) + \lim_{h \to 0} \left(\frac{p(x+h) - p(x)}{h}\right)$

$= p'(x) + q'(x)$

- To determine the derivative of a simple rational function, such as $f(x) = \frac{4}{x^6}$, express the function as a power, then use the power rule.
  If $f(x) = 4x^{-6}$, then $f'(x) = 4(-6)x^{(-6-1)} = -24x^{-7}$
- If you have a radical function such as $g(x) = \sqrt[3]{x^5}$, rewrite the function as $g(x) = x^{5/3}$, then use the power rule.
  If $g(x) = x^{5/3}$, then $g'(x) = \frac{5}{3} x x^{2/3} = \frac{5}{3} x \sqrt[3]{x^2}$

◉ 2.3 The Product Rule

1) $y = (2x)^2 \quad 4x^2 \rightarrow 8x$

2) $y = (2x+1)^2 \quad \cancel{4}x^2 + 2x + 2x + 1 = 4x^2 + 4x + 1 \rightarrow y' = 8x + 4$

3) $y = 3\sqrt{x} + \sqrt{x} + \frac{1}{x} + \frac{1}{x^2} \rightarrow x^{-2} \quad -2x^{-3}$

$-\frac{2}{x^3}$

$x^{1/3} = \frac{1}{3} x^{-\frac{2}{3}}$

$= \frac{1}{3\sqrt[3]{x^2}}$

$x^{1/2} = \frac{1}{2} x^{-1/2}$

$= \frac{1}{2\sqrt{x}}$

$\frac{-1}{x^2}$

$y' = \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{2\sqrt{x}} - \frac{1}{x^2} - \frac{2}{x^3}$

$(2x+1)^2$

↳ multiply the derivative by the slope of the inside.

ex. $y = (x^2+5x-1)^6$

$6(x^2+5x-1)^5 \times (2x+5)$

$= (12x+30)(x^2+5x-1)^5$

when $y = [g(x)]^n$, $\boxed{y' = n[g(x)]^{n-1}[g'(x)]}$

2-4, 5, 11, 18, 25

2. a) $f'(x) = 4$

b) $f'(x) = \frac{-1}{3\sqrt[3]{x^4}} - 2x$

c) $f'(x) = -2x+5$

d) $f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

e) $f'(x) = \frac{x^4}{16} = \frac{1}{16}x^4 = 4\left(\frac{1}{16 4}\right)x^3 = \frac{1}{4x^3}$

f) $-3x^{-4}$

3. a) $2x^2+3x+8x+12$

$2x^2+11x+12$

$4x+11$

b) $2(3)x^2 + 5(2)x - 4$

$6x^2+10x-4$

c) t^4-2t^3

$4t^3-2(3)t^2$

$4t^3-6t^2$

d) $\frac{1}{8}\times 8x^4 + \frac{1}{3}\times 3x^2 - \frac{1}{2}\times 2x$

x^4+x^2-x

e) $5x^8$

$5(8)x^7$

$40x^7$

4. a) $\frac{d}{dx}\times 3\times x^{5/3}$ $= \frac{5}{3}\times 3\times x^{2/3} = 5\sqrt[3]{x^2}$

b) $y' = \frac{d}{dx}4x^{-1/2} - \frac{d}{dx}\times\frac{6}{x} = {}^{2}4\times\left(\frac{-1}{2}\right)x^{-3/2}$ | $-6x^{-1} = (-6)(-1)x^{-2}$

$\frac{-2}{\sqrt[2]{x^3}} + \frac{6}{\sqrt{x}}$ $\qquad \frac{6}{\sqrt{x}}$

2.3 Product Rule

Feb. 20, 2013

Ex. 1 $y = (2x+5)(x^3+10)$

$= 2x^4 + 20x + 5x^3 + 50$

$= 8x^3 + 15x^2 + 20$

$y = (2)(x^3+10) + (2x+5)(3x^2)$ ← multiply opposites

$= 2x^3 + 20 + 6x^3 + 15x^2$

$= 8x^3 + 15x^2 + 20$

Proof for Product Rule

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $\quad g'(x) = \lim_{h \to 0} \frac{g(x+1) - g(x)}{h}$

$p'(x) = \lim_{h \to 0} \frac{p(x+h) + p(x)}{h}$

- If $p(x)$ is a combination of two functions, $p(x) = f(x)\,g(x)$ then $p'(x) = f'(x)(g(x)) + f(x)(g'(x))$

↑ opposites multiply.

PROOF

* $p'(x) = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$

- can add + subtract $f(x)g(x+h)$ $g(x)f(x+h)$
- $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$p'(x) = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$

$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x)}{h} + \lim_{h \to 0} \frac{f(x+h)g(x) - f(x)g(x)}{h}$

$= \lim_{h \to 0} f(x+h)\left[\frac{g(x+h) - g(x)}{h}\right] + \lim_{h \to 0} g(x)\left[\frac{f(x+h) - f(x)}{h}\right]$

$= f(x)g'(x) + g(x)f'(x)$ ▨

↑ means proof is done

Ex. a) • If $f(x) = x^2 - 3x(x+1)^3$, find the derivative using product rule.

b) $f(x) = (3x^2 - 10)^5(x)$

a) $f' = (2x-3)(x+1)^3 + (x^2-3x)(3)(x+1)^2$

b) $f' = 5(3x^2-10)^4(6x)(x) + (3x^2-10)^5(1)$

$f' = (x+1)^2\left[(2x-3)(x+1) + (x^2-3x)(3)\right]$

$= (x+1)^2(2x^2+2x-3x-3+3x^2-9x)$

$= (x+1)^2(5x^2-10x-3)$

$(3x^2-10)^4\left[(30x^2+3x^2-10)\right]$

$= (3x^2-10)^4(33x^2-10)$

Example $\quad y = \underbrace{(3x^2-1)^{10}}_{f}\underbrace{(x^3+5x)^6}_{g}$

$f' \rightarrow 60x(3x^2-1)^9$

$g' \rightarrow 6(x^3+5x)^5(3x^2+5)$

$y' = 60x(3x^2-1)^9(x^3+5x)^6 + (3x^2-1)^{10}(6)(x^3+5x)^5(3x^2+5)$

$y' = 6(3x^2-1)^9(x^3+5x)^5\left[10x(x^3+5x) + (3x^2-1)(3x^2+5)\right]$

$y' = 6(3x^2-1)^9(x^3+5x)^5\left[10x^4+50x^2+9x^4-3x^2+15x^2-5\right]$

$y' = 6(3x^2-1)^9(x^3+5x)^5(19x^4+62x^2-5)$