2.4 The Quotient Rule

The Quotient Rule

If $h(x) = \frac{f(x)}{g(x)}$, then $h'(x) = \frac{f'(x) * g(x) - f(x)g'(x)}{[g(x)]^2}$

$g(x) \neq 0$

In Leibniz notation, $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$

- The derivative of a quotient of 2 differentiable functions is not the quotient of their derivatives.
- The quotient rule for differentiation:

  If $h(x) = \frac{f(x)}{g(x)}$, then $h'(x) = \frac{f'(x)(g(x)) - g'(x)(f(x))}{[g(x)]^2}$

  $g(x) \neq 0$

- To find the derivative of a rational function →

$f(x) = \frac{x-2}{1+x}$ ① Leave the function in fraction form, use the quotient rule

$f(x) = (x-2) \times (1+x)^{-1}$ ② Express the function as a product, and use the product and power of a function rules.

The Derivative of Composite Functions (2.5)

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- You can find derivative and combine it or vice versa

$y = (2x+10)^9$
$y' = 9(2x+10)^8 \times 2$

$y = u^9 \quad u = 2x+10$
$y' = 9u^8 \cdot u'$
$= 9(2x+10)^8(2)$

$$\left[\begin{array}{l} \dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} \\ y'(x) = y'(u) \times u'(x) \end{array}\right] \rightarrow \text{CHAIN RULE}$$

$y = f(g(x))$
$y' = f'(g(x)) \times g'(x)$

ex. $f(x) = x^{10} + x^4 \qquad g(x) = \sqrt{x}$

$$\left.\begin{array}{l} y = f(g(x)) = (\sqrt{x})^{10} + (\sqrt{x})^4 \\ \qquad = x^5 + x^2 \\ y' = 5x^4 + 2x \end{array}\right] \text{- composed, then derivative}$$

$f'(x) = 10x^9 + 4x^3$
$g'(x) = \dfrac{1}{2\sqrt{x}}$

$y' = f'(\sqrt{x}) \cdot \dfrac{1}{2\sqrt{x}}$
$= \left[10(\sqrt{x})^9 + 4(\sqrt{x})^3\right] \cdot \dfrac{1}{2\sqrt{x}}$
$= 5(\sqrt{x})^8 + 2(\sqrt{x})^2$
$= 5x^4 + 2x$

so, $y = (f \circ g \circ h)(x)$
$= f(g(h(x)))$
$= f'(g(h(x))) \times g'(h(x)) \times h'(x)$

13a $y = 3u^2 - 5u \quad u = x^2 - 1 \quad x = 2$

$$
\begin{aligned}
f' &= y'(u(x)) \times u'(x) \\
&= 6(x^2-1) - 5 \times 2x \\
&= (6x^2 - 6 - 5) \times 2x \\
&= (6x^2 - 11) \times 2x \\
&= 52
\end{aligned}
$$

7. $h'(x) = 2\left(\frac{1}{x} - 3\right) \times -\frac{1}{x^2}$

$$
\begin{aligned}
&= \left(\frac{2}{x} - 6\right) \cdot -\frac{1}{x^2} \\
&= \frac{-2}{x^3} + \frac{6x}{x^3} \\
&= \frac{-2 + 6x}{x^3}
\end{aligned}
$$

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$y = (6x-5)^5 (x^2+7)^8$

$y' = (6x-5)^5\, 8(x^2+7)^7\, 2x + (x^2+7)^8\, 5(6x-5)^4\, 6$

$= 2(6x-5)^4 (x^2+7)^7 \left[(6x-5)(8x) + (x^2+7)(15)\right]$

$2(6x-5)^4 (x^2+7)^7 \left[48x^2 - 40x + 15x^2 + 105\right]$

$2(6x-5)^4 (x^2+7)^7 (63x^2 - 40x + 105)$

$y = \dfrac{x^3}{(x^2-8)^2}$

$= \dfrac{3x^2(x^2-8)^2 - 2(x^2-8)2x(x^3)}{(x^2-8)^4}$

$= \dfrac{x^2(x^2-8)\left[3(x^2-8) - 4x^2\right]}{(x^2-8)^4} \rightarrow 3x^2 - 24 - 4x^2$

$-x^2 - 24$

$= \dfrac{-x^2(x^2+24)}{(x^2-8)^3}$

$y = \left(\sqrt[5]{2x+3}\right)(5x+1)^2$

$= (2x+3)^{1/5}(5x+1)^2$

$= \frac{1}{5}(2x+3)^{-4/5}(2)(5x+1)^2 + (2x+3)^{1/5}(2)(5x+1)(5)$

$= \frac{2}{5}(2x+3)^{-4/5}(5x+1)\left[(5x+1) + 25(2x+3)\right]$

$= \frac{2}{5}(2x+3)^{-4/5}(5x+1)\left[5x+1+50x+75\right] \rightarrow \frac{1}{5}\left(\frac{-4}{5}\right)$

$= \frac{2}{5}(2x+3)^{-4/5}(5x+1)\left[55x+76\right]$

$y = \left(\dfrac{x-1}{x^2+1}\right)^3$

$y = \dfrac{(x-1)^3}{(x^2+1)^3}$ $= \dfrac{3(x-1)^2(x^2+1)^3 - 3(x^2+1)^2(x-1)^3}{(x^2+1)^6}$

$= \dfrac{3(x^2+1)^2(x-1)^2\left[(x^2+1) - 2x(x-1)\right]}{(x^2+1)^6}$

$= \dfrac{3(x^2+1)^2(x-1)^2(x^2+1-x+1)}{(x^2+1)^6}$

$= \dfrac{3(x-1)^2(-x^2+2x+1)}{(x^2+1)^4}$

$\dfrac{x-1}{x^2+1} \rightarrow \dfrac{(x^2+1)-(x-1)(2x)}{(x^2+1)^2} = \dfrac{x^2+1-2x^2+2x}{(x^2+1)^2}$

$= \dfrac{-x^2+2x+1}{(x^2+1)^2}$

$3\dfrac{(-x^2+2x+1)}{(x^2+1)^2}\dfrac{(x-1)^2}{(x^2+1)^2} \rightarrow \dfrac{3(-x^2+2x+1)(x-1)^2}{(x^2+1)^4}$