## 2.5 The Derivatives of Composite Functions

- composition → combining 2 functions as $f(g(x))$

### Definition of a Composite Function

Given 2 functions $f$ and $g$, the composite function $(f \circ g)$ is defined by $(f \circ g)(x) = f(g(x))$

### The Chain Rule

If $f$ and $g$ are functions that have derivatives, then the composite function $h(x) = f(g(x))$ has a derivative given by $h'(x) = f'(g(x))g'(x)$

### The Chain Rule in Leibniz Notation

If $y$ is a function of $u$ and $u$ is a function of $x$ (so that $y$ is a composite function), then $\frac{dy}{dx} = \frac{dy}{du}\left(\frac{du}{dx}\right)$

provided that $\frac{dy}{du}$ and $\frac{du}{dx}$ exist.

### Power of a Function Rule

If $n$ is a real number and $u = g(x)$, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

or $$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x).$$

Leibniz Notation → $$\frac{d[h(x)]}{dx} = \frac{d[f(g(x))]}{d[g(x)]} \cdot \frac{d[g(x)]}{d(x)}$$

### Power of a Function Rule

$$\frac{d}{dx}[g(x)]^n = \frac{d[g(x)]^n}{d[g(x)]} \cdot \frac{d[g(x)]}{dx}$$
$$= n[g(x)]^{n-1} \cdot g'(x)$$