⊙ 3.2 Closed Interval Method

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given $(\sin x)' = \cos x$
$(\cos x)' = -\sin x$

Find $(\tan x)'$

$\hookrightarrow \left(\frac{\sin x}{\cos x}\right)' \rightarrow \frac{\cos x \sin x' - \cos x' \sin x}{\cos^2 x}$

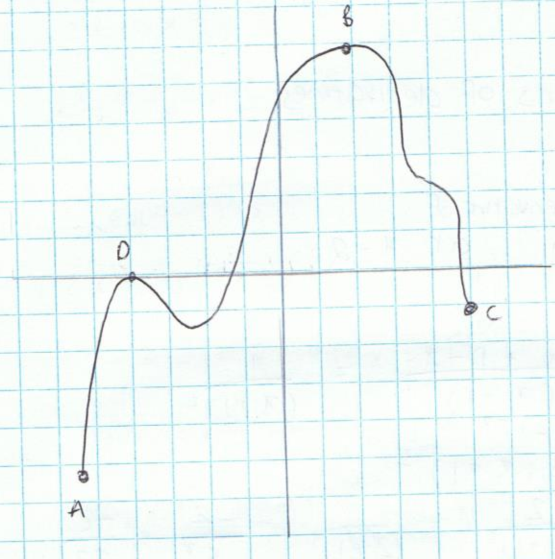
$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

ex. $y = 5(\sin^2 x + \cos^2 x) + \frac{1}{x}$

$\hookrightarrow 1$



A - absolute/global min
B - " max
C - endpoint (can't be local, is not global)
D - local max

$f(x)$ on $x \in [a, b]$

① Set $f'(x) = 0$ solve for x, that only belongs to interval
↳ critical number

② Find y-values for x-values from step 1
Find y-values at endpoints ($f(a)$ and $f(a)$)

③ Highest value from step 2 is global max,
Lowest is global min

Example 1 → $f(x) = x^3 - 3x^2 + 1$ $x \in [-1/2, 4]$

$3x^2 - 6x = 0$
$3x(x - 2) = 0$
$x = 0, 2$

$f(0) = 1$
$f(2) = -3$ ← global min
$f(-1/2) = 1/8$
$f(4) = 17$ ← global max

· Use second derivative to find point of inflection

Example - $f(x) = \frac{4x}{x^2+1}$

$x \in [2,4]$

$$f'(x) = \frac{4x^2 + 4 - 8x^2}{(x^2+1)^2}$$

$$((x^2+1)^2)\,0 = \frac{-4(x^2-1)}{(x^2+1)^2}$$

-4

$x = \pm 1 \notin [2,4]$

$f(4) = 16/17$ ← global min

$f(2) = 8/5$ ← global max

3.2 Maximum and Minimum on an Interval (Extreme Values)

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- The maximum and minimum values of a function on an interval are also called extreme values, or absolute extrema
- The maximum value of a function that has a derivative at all points in an interval occurs at a "peak" ($f'(c) = 0$) or at an endpoint of the interval, $[a, b]$.
- The minimum value occurs at a "valley" ($f'(c) = 0$) or at an endpoint of the interval, $[a, b]$.

Algorithm for Finding Extreme Values

For a function $f(x)$ that has a derivative at every point in an interval $[a,b]$, the maximum or minimum value's can be found by using the following procedure:

1) Determine $f'(x)$. Find all points in the interval $a \le x \le b$, where $f'(x) = 0$.

2) Evaluate $f(x)$ at the endpoints a and b, and at points where $f'(x) = 0$.

3) Compare all the values found in step 2.
   - The largest of these values is the maximum value of $f(x)$ on the interval $a \le x \le b$.
   - The smallest of these values is the minimum value of $f(x)$ on the interval $a \le x \le b$.