3.3 Optimization Problems

Mar. 5, 2014

pg. 141

Ex. 1 Determine the max area if 400 m fencing is available for a rectangular field.

$A = l \times w$

$P = 2(l + w)$

$\frac{400}{2} = l + w = 200$

$l = 200 - w$

$A = 200w - w^2$

$A' = -2w + 200$

$0 = -2w + 200$

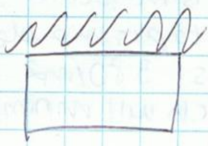
$\frac{-200}{-2} = w$

$w = 100$

max area is always a square

$l = 100$

ex. 2



$A = (400 - 2w)w$

$P = l + 2w$

$400 - 2w = l$

$A = 400w - 2w^2$

$A' = -4w + 400$

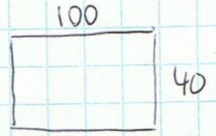
$w = 100$

$P = l + 2(100)$

$l = 200$

If one side is covered, the ratio will be 1:2 for l and w to maximize area.

ex. 3



Max volume?

$V = l \times w \times h$ $\qquad$ $l = 100 - 2x$

$h = x$

$V = (100-2x)(x)(40-2x)$ $\qquad$ $w = 40 - 2x$

$= (100x - 2x^2)(40 - 2x)$

$= 4000x - 80x^2 - 200x^2 + 4x^3$

$0 = (4)(3x^2) - 280(2x) + 4000$

$0 = 12x^2 - 560x + 4000$

$= 4(3x^2 - 140x + 1000)$

$$x = \frac{140 \pm \sqrt{140^2 - 4(1000)(3)}}{6}$$

$x \doteq 38.9, 8.8$

$\hookrightarrow$ inadmissible

$l = 82.4$ cm

$w = 22.4$ cm

$h = 8.8$ cm

$V = 16243\ cm^3$

$100 - 2x > 0$ $\qquad$ $40 - 2x > 0$

$100 > 2x$ $\qquad$ $20 > x$

$50 > x$

ex. 4 A cylindrical storage tank with a capacity of $1000 m^3$ is to be made. The base is made of steel steel that costs \$100/m^2. The top is made of steel that costs \$50/m^2. And the wall is \$80/m^2. Determine the radius and the height 'h' which will minimize costs.

Volume $= \pi r^2 h$

$\frac{1000}{\pi r^2} = h$

Total cost $= (100\pi r^2) + (80(2\pi r h)) + 50(\pi r^2)$

$= 150\pi r^2 + 160\pi r h$

$= 150\pi r^2 + 160\cancel{\pi r}\left(\frac{1000}{\pi r^2}\right)$

$= 150\pi r^2 + 160000 r^{-1}$

$A' = 150\pi(2r) - 160000 r^{-2}$

$= \frac{300\pi r^3}{r^2} - \frac{160000}{r^2}$

$$0 = 300\pi r^3 - 160000$$

$$\sqrt[3]{\frac{160000}{300\pi}} = r$$

$$r = 5.5 \text{ cm}$$

3.3 Optimization Problems

Mar. 14. 2014

Pg. 141

- In an optimization problem, you must determine the maximum or minimum value of a quantity
- An optimization problem can be solved using a mathematical model that is developed using information given in the problem. The numerical solution represents the extreme value of the model

- Algorithm for Solving Optimization Problems

1) Understand the problem, and identify quantities that can vary. Determine a function in one variable that represents the quantity to be optimized.
2) Whenever possible, draw a diagram, labelling the given and required quantities.
3) Determine the domain of the function to be optimized, using the information given in the problem
4) Use the algorithm for extreme values to find the absolute maximum or minimum value in the domain
5) Use your result for step 4 to answer the original problem