4.1 Increasing and Decreasing Functions

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- from left to right function rises $f' > 0$
- from left to right function falls $f' < 0$

ex. 1

$y = 2x + 3$

$y' = 2$

$y' > 0$ for all values of x

$\Rightarrow f(x)$ is always increasing

ex. 2

$y = 5$

$y' = 0$

horizontal line.

ex. 3 $y = x^2$

$y' = 2x$

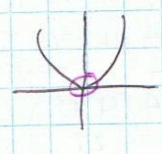
$= 0$, if $x = 0$

	$(-\infty, 0)$	$(0, \infty)$
$2x$	$-$	$+$
	$-$	$+$

$f(x)$ decreases on the interval $x \in (-\infty, 0)$

$f(x)$ increases on the interval $x \in (0, \infty)$

$(0,0)$ is therefore a minimum



Steps to find intervals of increase + decrease

1) Find derivative
2) Find the derivative $= 0$, or dne
3) Make the chart using the values in step 2 to find inc./dec. intervals

$-, + \rightarrow$ min $\cup$

$+, - \rightarrow$ max. $\cap$

Example 4 - $F(x) = x^3 + 3x^2 - 2$

(find $\uparrow, \downarrow$, max, min)

$f'(x) = 3x^2 + 6x -$

$= 3x(x+2)$

$0, -2$

	$-\infty, -2$	$-2, 0$	$0, \infty$
x	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$
	$+$	$-$	$+$

-2 0

$0 \rightarrow -2$

$-2 \rightarrow 2$

inc $\rightarrow x \in (-\infty, -2) \cup (0, \infty)$

dec $\rightarrow x \in (-2, 0)$

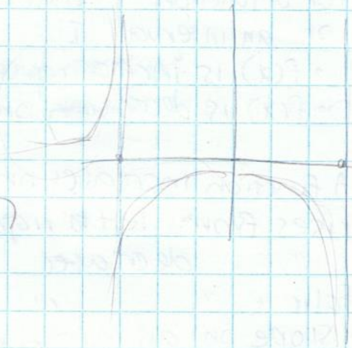
• no global max/min

(max $\rightarrow (-2, 2)$)

(min $\rightarrow (0, -2)$)

Example 5

$f(x) = \frac{1}{x^2-9}$



$f'(x) = \frac{d}{dx}(x^2-9)^{-1}$

$= -1(x^2-9)^{-2}(2x)$

$0 = \frac{-2x}{(x^2-9)^2}$

$x = 0, \pm 3$

	$(\infty,-3)$	$(-3,0)$	$(0,3)$	$(3,\infty)$
$-2x$	+	+	−	−
$(x+3)^2$	+	+	+	+
$(x-3)^2$	+	+	+	+
$f'(x)$	+	+	−	−

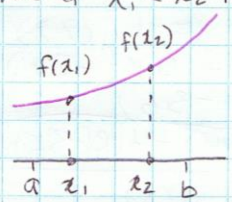
VA $x=-3$ $x=0$ VA $x=3$

$f(x)$ inc $\rightarrow$ $x \in (\infty, -3) \cup (-3, 0)$

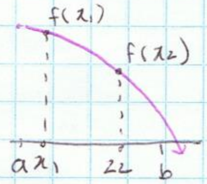
dec $\rightarrow$ $x \in (0, 3) \cup (3, \infty)$

local max $(0, -1/9)$

- A function f is increasing on an interval if, for any value of $x_1 < x_2$ in the interval $f(x_1) < f(x_2)$



- A function f is decreasing on an interval if, for any value of $x_1 < x_2$ in the interval, $f(x_1) > f(x_2)$



- For a function f that is continuous and differentiable on an interval I
  - $f(x)$ is increasing on I if $f'(x) > 0$ for all values of x in I
  - $f(x)$ is decreasing on I if $f'(x) < 0$ for all values of x in I

- A function increases on an interval if the graph rises from left to right
- " decreases " " " falls " ' ' '
- Slope of a point on an increasing curve is always (+)
- " a decreasing curve is always (−)

⓪ Practice

1\. a) $f'(x) = 3x^2 + 12x$

$0 = 3x(x+4)$

$x = 0, -4$

b) $f'(x) = (x^2+4)^{1/2}$

$= \frac{1}{2}(x^2+4)^{-1/2}(2x)$

$0 = \frac{x}{(\sqrt{x^2+4})}$

$x = 0$

c) $f'(x) = 2(2x-1)(2)(x^2-9) + (2x-1)^2(2x)$

$= (2x-1)(4x^2-36 + 4x^2 - 2x)$

$0 = (2x-1)(8x^2-2x-36)$

$x = 1/2$ $\quad x = \frac{2 \pm \sqrt{(4) - 4(8)(-36)}}{2(8)}$

$= \frac{2 \pm 34}{16}$

$= \frac{1 \pm 17}{8}$

$= 2.25$

$= -2$

d)

$f(x) = x^3 - 12x$

a) Find the intervals of increase and decrease
b) Classify the critical points as local max or local min
c) Find the x-intercepts
d) Graph it

a) $f(x) = x^3 - 12x$
$f'(x) = 3x^2 - 12$
$x = \pm 2$

	$-\infty, -2$	$-2, 0$	$0, 2$	$2, \infty$
$3x^2-12$	+	−	−	+
	+	−	−	+

inc → $x \in (-\infty, -2) \cup (2, \infty)$
dec → $x \in (-2, 0) \cup (0, 2)$

b) $f(-2) = 16$ → local max
$f(0) = 0$
$f(2) = -16$ → local min

c) $f(x) = (x)(x^2 - 12)$ $\qquad F'(x) = 3x^2 - 12$
$x = 0$ $\qquad x = \pm 2$
$x = \pm 2\sqrt{3}$

d)

Find the critical point of $f(x) = \frac{x}{x^2-1}$

$f(x) = x(x^2-1)^{-1}$

$f'(x) = (x^2-1)^{-1} + (x^2-1)^{-2}(-1)(x)(2x)$

$= \frac{1}{(x^2-1)} \quad \frac{-2x^2}{(x^2-1)^2}$

$= \frac{(x^2-1)-2x^2}{(x^2-1)^2}$

$= \frac{-x^2-1}{(x^2-1)^2}$