4.5 An Algorithm for Curve Sketching

Mar. 23. 2014

: The first + second derivatives of a function give info about the shape of the graph of the function

Sketching the Graph of a Polynomical or Rational Function

1) Use the function to
   - determine the domain + any discontinuities
   - determine intercepts
   - find asymptotes + determine function behaviour relative to asymptotes

2) Use the first derivative to $(f'(x))$
   - find the critical numbers
   - determine where the function is increasing + where it is decreasing
   - identify local max. and min.

3) Use the second derivative to $(f''(x))$
   - determine where the graph is concave up and where it's concave down
   - find POIs

→ $f''(x)$ can be used to identify local max or min

4) Calculate values of y that correspond to critical points and POI. Use info to sketch graph.

*not all steps are needed in all equations.

$$f(x) = \frac{x^2}{(x-1)^2}$$

→ x, y-int
→ HA, VA, behaviour
→ intervals of inc/dec
→ classify critical points
→ domain/range
→ intervals of ~~CA~~ CU/CD
→ any POIs → graph

10e) $\frac{x}{x^2-4x+4}$

$\frac{x}{(x-2)(x-2)}$ $(0,0)$ $= \frac{x}{(x-2)^2}$

$x = 2$
$y = 0$

$x \to 2^+ \quad y \to \infty$
$2^- \quad y \to +\infty$
$x \to \infty \quad y \to 0^+$
$-\infty \quad y \to 0^-$

$$f''(x) = \frac{(x-2)^2 - x(2)(x-2)}{(x-2)^4}$$
$$= \frac{(x-2)(x-2-2x)}{(x-2)^4}$$
$$= \frac{-(x+2)}{(x-2)^3}$$
$x = -2, 2$

$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
$-$	$+$	$-$

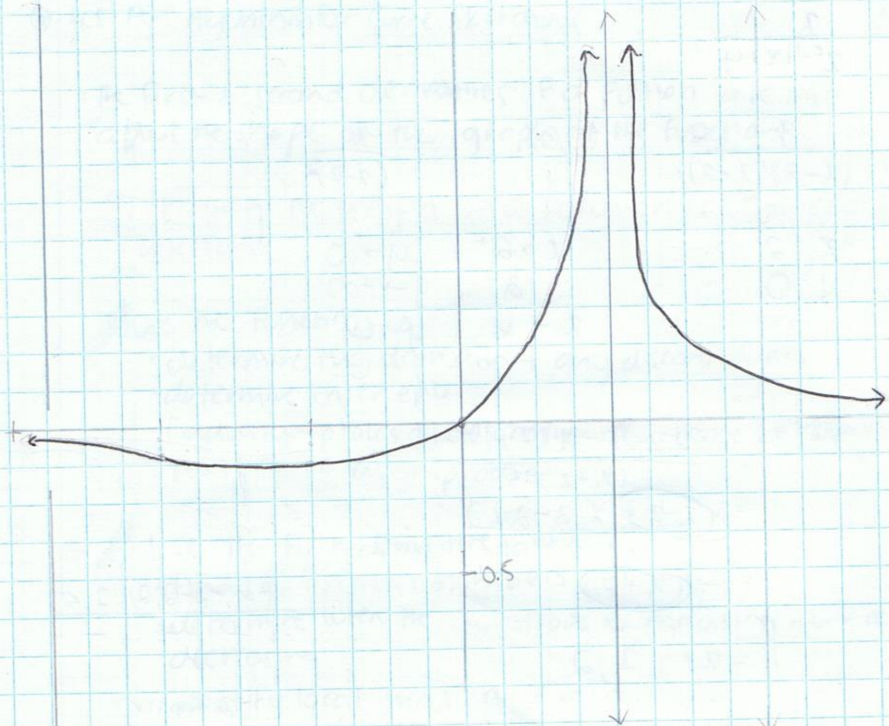
$-2 \to$ min
$2 \to$ max

$$f''(x) = \frac{-(x-2)^3 - (-x+2)(3)(x-2)^2}{(x-2)^6}$$
$$= \frac{(x-2)^2[-x+2-(-3x-6)]}{(x-2)^6}$$
$$= \frac{(x-2)^2[-x+2+3x+6]}{(x-2)^6}$$
$$= \frac{(2x+8)}{(x-2)^4}$$
$$= \frac{2(x+4)}{(x-2)^4}$$
$x = 2, -4$

$-\infty,-4$	$-4,\infty$
$-$	$+$

$-4 \to$ POI

$(-2, -0.125)$
$(2$, undefined$)$
$(-4, -0.11)$
$(0, 0)$



10f)

$$\frac{t^2-3t+2}{t-3}$$

$$\begin{array}{r|rrr} 3 & 1 & -3 & 2 \\ & \downarrow & 3 & 0 \\ \hline & 1 & 0 & 2 \end{array}$$

$$\begin{array}{r|l} & t+0 \\ t-3 & t^2-3t+2 \\ & t^2-3t \\ \hline & 0t+2 \\ & 0t+0 \\ \hline & 2 \end{array}$$

$y=t$
$x=3$

$x \to 3^+ \quad y \to \infty$
$x \to 3^- \quad y \to -\infty$

$(2,0)$
$(1,0)$
$(0,-2/3)$

$$f'(x) = \frac{(2t-3)(t-3)-(t^2-3t+2)}{(t-3)^2}$$

$$= \frac{\cancel{2t^2}\cancel{-3t}-6t+9\cancel{-t^2}\cancel{+3t}-2}{(t-3)^2}$$

$$= \frac{t^2-6t+7}{(t-3)^2} = \frac{(t-7)(t+1)}{(t-3)^2} \quad 7,-1,3$$

$-\infty,-1$	$-1,3$	$3,2$	$7,\infty$
+	+	−	+