5.1 Derivatives of Exponential Functions $y=e^x$

- cannot by $y' = xe^{x-1}$
  ↳ b/c that only works if the variable is the base

So, $y = e^x$ y'?

$a^{m+n} = a^m a^n$

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h} - x^x}{h}$$

$$= \lim_{h\to 0} \frac{e^x e^h - e^x}{h} = \lim_{h\to 0} e^x \left[\frac{e^h - 1}{h}\right] =$$

h	$\frac{e^h - 1}{h}$
0.1	1.0517
0.01	1.005017
0.001	1.00050017
0.0001	1.0000500017
0.00001	1.000005000017
$h \to 0$	1

$y' = e^x$
↳ euler's number

$y = e^x$	$y' = e^x$
$y = e^{g(x)}$	$y' = e^{g(x)} \cdot g'(x)$
$y = e^{\square}$	$y' = e^{\square} \cdot \square'$

$\square^2 \to 2\square \cdot \square'$

ex. $y = e^{x^2}$ $y' = e^{x^2} \cdot 2x$

ex. $y = 3e^x$ $y' = 3e^x$

$y = e^{10x}$ $y' = e^{10x} \cdot 10$

$y = e^{10}$ $y' = e^{10} \cdot 0$
$y' = 0$

ex. 2 $y = \sqrt{x}\, e^{x}$

$y' = \frac{d}{dx} e^{x} \cdot \sqrt{x}$

$y' = \frac{1}{2\sqrt{x}} e^{x} + e^{x}\sqrt{x}$

$= \frac{e^{x}}{2\sqrt{x}} + \frac{2e^{x}\sqrt{x}\sqrt{x}}{2\sqrt{x}}$

$= \frac{2xe^{x}}{2\sqrt{x}}$

$= \frac{xe^{x}}{\sqrt{x}}$

ex. 3 $y = e^{\left(\frac{1}{x+}\right)}$ $y' = e^{\frac{-1}{x+1}} \cdot \frac{1}{(x+1)^2}$

ex 4. Find the eq^n of tangent when $x=0$ for $f(x) = \frac{e^{2x}}{1+e^{x^2}}$

m = ~~——~~ derivative → quotient rule

$x = 0$

$y = 1/2$

$f'(x) = \frac{2e^{2x}(1+e^{x^2}) - (e^{2x})(2xe^{x^2})}{(1+e^{x^2})^2}$

$= \frac{e^{2x}(2 + 2e^{x^2} - \cancel{2xe^{x^2}})}{(1+e^{x^2})^2}$

when $x = 0$

$= \frac{1(2 + 2(1))}{(1+1)^2}$

$= \frac{4}{4}$

$= 1$

$y = mx + b$

$\frac{1}{2} = 1(0) + b$

$b = \frac{1}{2}$

$y = x + 1/2$

Derivative of $f(x) = e^x$

For the function $f(x) = e^x$, $f'(x) = e^x$; $\frac{d}{dx}(e^x) = e^x$

Derivative of a Composite Function Involving e^x

In general, if $f(x) = e^{g(x)}$,
$f'(x) = e^{g(x)} \cdot g'(x)$ by the chain rule.

$$\frac{d}{dx}(e^{g(x)}) = \frac{d(e^{g(x)})}{d(g(x))} \cdot \frac{d(g(x))}{dx}$$

- the slope of the tangent at a point on the graph of $y = e^x$ equals the value of the function at this point

- The rules for differentiating functions, such as the product, quotient and chain rules, also apply to combinations involving exponential functions of the form $f(x) = e^{g(x)}$

- e is called Euler's number or the natural number, where $e \doteq 2.718$.

Practice

pg. 232 1. because x is not a base, but instead is an exponent.

2. a) $y' = 3e^{3x}$
   b) $y' = 3e^{3t-5}$
   c) $y' = 20e^{10t}$
   d) $y' = -3e^{-3x}$
   e) $y = (2x-6)e^{5-6x+x^2}$
   f) $y = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

3. a) $y' = 6x^2e^{x^3}$
   b) $y' = 3xe^{3x}$
   c) $y' = \frac{-3x^3e^{-x^3} - xe^{-x^3}}{x^2}$
   $= \frac{-(3x^2-1)(e^{-x^3})}{x}$
   d) $y' = \frac{e^x}{2\sqrt{x}} + e^x\sqrt{x}$
   $= e^x\left(\frac{1}{2\sqrt{x}} + \sqrt{x}\right)$
   e) $h'(t) = 2te^{t^2} - 3e^{-t}$
   f) $g'(t) = \frac{2e^{2t}(1+e^{2t}) - (2e^{2t})(e^{2t})}{(1+e^{2t})^2}$