5.2 The derivative of the General Function, $y=b^x$

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$y = e^x \quad y' = e^x$

ex. $y = 5^x \quad y' = 5^x \cdot \ln 5$

$\frac{a^h - 1}{h} = \ln a$

$y = a^x \quad y' = a^x \cdot \ln a$

ex. $y = 15^{x^2}$

$y' = 15^{x^2} \cdot \ln 15 \cdot 2x$

ex. $y' = (5^{x^2-1})(x^3)$

$= 3x^2(5^{x^2-1}) + (x^3)(\ln 5)(2x)(5^{x^2-1})$

$= 5^{x^2-1}(x^2)(3 + 2x^2 \ln 5)$

ex. $y = \frac{\sqrt[3]{10^x}}{x^5}$

$y' = \frac{10^{x/3}(\ln 10)(1/3)(5x^4) - (x^5)(10^{x/3})}{x^{10}}$

$= \frac{(10^{x/3})(\cancel{x^4})[\frac{5}{3}\ln 10 - x]}{x^{\cancel{10}}}$

$= \frac{(10^{x/3})[5/3 \ln 10 - x]}{x^6}$

8. a) $e^{3x+2} = 5$

$\log_e 5 = 3x + 2$

$x = -0.13$

$\ln e^{3x+2} = \ln 5$

$(3x+2)\ln e = \ln 5$

$\ln e = \cancel{x}$

$x = \frac{\ln 5 - 2}{3}$

b) $y = (e^{1/\sqrt{x}})(x^2)$

$y' = 2x(e^{1/\sqrt{x}}) + (x^2)(-\frac{1}{2}x^{-3/2})(e^{1/\sqrt{x}})$

$\frac{1}{\sqrt{x}} = x^{-1/2} = -\frac{1}{2}x^{-3/2}$

$= 2x(e^{1/\sqrt{x}}) + (-1/2\, x^{1/2})(e^{1/\sqrt{x}})$

$= (e^{1/\sqrt{x}})x^{1/2}[2x^{1/2} - 1/2]$

c) $y = \frac{e^{2x}}{(x+1)^{10}}$

$y' = \frac{2e^{2x}(x+1)^{10} - 10(x+1)^9(e^{2x})}{(x+1)^{20}}$

$= \frac{e^{2x}(x+1)^9[2(x+1) - 10]}{(x+1)^{20}}$

$= \frac{e^{2x}\cancel{(x+1)^9}(2x-8)}{(x+1)^{\cancel{20}\,11}}$

$= \frac{e^{2x}(2x-8)}{(x+1)^{11}}$

$= \frac{2(e^{2x})(x-4)}{(x+1)^{11}}$

When is the tangent horizontal?

$e^{2x} \neq 0$ $\quad$ $0 = x - 4$

$x = 4$

ex. $e^{2x} - 1 = 0$

$e^{2x} = 1$

$\ln(2x) = \ln 1$

$2x = 0$

$x = 0$

5.2 The Derivative of the General Exponential Function, $y=b^x$

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Derivative of $f(x)=b^x$

$\lim_{n\to 0} \frac{b^n-1}{n} = \ln b$ and if $f(x)=b^x$, then $f'(x)=(\ln b)\times b^x$

Derivative of $f(x)=b^{g(x)}$

For $f(x)=b^{g(x)}$, $f'(x)=b^{g(x)}(\ln b)(g'(x))$

- If $f(x)=b^x$, then $f'(x)=b^x(\ln b)$
  In Leibniz notation, $\frac{d}{dx}(b^x)=b^x(\ln b)$
- If $f(x)=b^{g(x)}$, then $f'(x)=b^{g(x)}(\ln b)(g'(x))$
  In Leibniz notation, $\frac{d}{dx}(b^{g(x)}) = \frac{d(b^{g(x)})}{d(g(x))} \times \frac{d(g(x))}{dx}$
- $\lim_{n\to 0} \frac{b^n-1}{n} = \ln b$
- when you are differentiating a function that involves an exponential function, use the rules given above, along with the sum, difference, product, quotient, and chain rules as required.

Practice

1. a) $y' = \ln 2(2^{3x})(3)$
   b) $y' = 3.1^x(\ln 3.1) + 3x^2$
   c) $s' = (10^{3t-5})(\ln 10)(3)$
   d) $w' = (10^{n^2-6n+5})(\ln 10)(2n-6)$
   e) $y' = (3^{x^2+2})(\ln 3)(2x)$
   f) $y' = 400(2^{x+3})(\ln 2)$