

5.3 Optimization Problems Involving Exponential Functions

Apr. 4, 2013

- Optimizing means determining the values of the independent variable so that the values of a function that models a situation can be minimized or maximized.
- The techniques used to optimize an exponential function model are the same as those to optimize polynomial and rational functions.
- Apply the algorithm introduced in Chapter 3 to solve an optimization problem:
 - Understand the problem, and identify quantities that can vary. Determine a function in ^{one} variable that represents the quantity to be optimized.
 - Determine the domain of the function to be optimized, using the information given in the problem.
 - Use the algorithm for finding extreme values (from Ch. 3) to find the absolute max or min values of the function on the domain.
 - Use the result from step 3 to answer the original problem.
 - Graph the original function using tech. to confirm results.

Practice

1. —

$$\begin{aligned}
 2. \ a) \quad f'(x) &= -e^{-x} - (-3)e^{-3x} \\
 0 &= 3e^{-3x} - e^{-x} \\
 &= e^{-3x} (3 - e^{2x}) \\
 e^{-x} &= 0 \quad -e^{2x} = -3 \\
 e^{2x} &= 3 \\
 \ln e^{2x} &= \ln 3 \\
 \frac{2x}{2} &= \frac{\ln 3}{2}
 \end{aligned}$$

$$\begin{aligned}
 f(0) &= 0 \quad \leftarrow \text{min} \\
 f(10) &= 4.53 \times 10^{-5} \\
 f\left(\frac{\ln 3}{2}\right) &= 0.384 \quad \leftarrow \text{max}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad m(x) &= 1(e^{-2x}) + (x+2)(-2)e^{-2x} & f(-4) &= -5961.92 \quad \leftarrow \text{min} \\
 &= e^{-2x} (1 + (x+2)(-2)) & f(3/2) &= 10.04 \quad \leftarrow \text{max} \\
 &= e^{-2x} (1 - 4 - 2x) & f(4) &= 2.012 \times 10^{-2} \\
 &= e^{-2x} (-2x - 3) \\
 &\quad \frac{\ln 3}{2}
 \end{aligned}$$