5.4 Notes

$\sin(a+b) = \sin a \cos b + \sin b \cos a$

$\boxed{y = \sin x \qquad y' = \cos x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cosh + \sinh \cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cosh - \sin x}{h} + \frac{\sinh \cos x}{h}$$

$$= \lim_{h\to 0} \sin\left[\frac{(\cos h - 1)}{h}\right] + \cos x \left[\frac{\sin h}{h}\right]$$

* get limit using table

h	$\frac{\sin h}{h}$	$\frac{\cos h - 1}{h}$
0.1	0.99	−0.04
0.01	0.999	−0.0004
0.001	0.9999	−0.00004
$h \to 0$	1	0

$$= \lim_{h\to 0} \sin x [0] + \cos x [1]$$

$\boxed{y' = \cos x}$

$y = \cos x \quad y' = -\sin x$

$y = \sin \square \quad y' = \cos \square \times \square'$

$y = \cos \square \quad y' = -\sin \square \times \square'$

$y = \sin x \quad y' = \cos x$

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ex. $f(x) = \sin(1/x)$

$f'(x) = \cos(1/x) \times (-1/x^2)$

ex. $\lim_{h \to 0} \frac{\sin 3h}{h} \rightarrow \frac{3 \sin 3h}{3h} = 3$

b/c $\frac{\sin 3h}{3h} = 1$ as $\frac{\sin h}{h} = 1$

ex. $\lim_{h \to 0} \frac{\sin 3h}{4h} = \frac{3 \sin 3h}{3 \times 4h} = \frac{3 \sin 3h}{4 (3h)} = \frac{3}{4}$

Locitanal's Rule

when $\frac{\sin 3h}{h} = \frac{0}{0}$

find y'

now plug in 0

you can do $\rightarrow \frac{3\cos 3h}{1} \longrightarrow \frac{3\cos 3(0)}{1} = 3$

ex. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \frac{0}{0}$ so, $\frac{2x}{1} \Rightarrow \frac{2(1)}{1} = 2$

5.4 - The Derivative of $y = \sin x$ and $y = \cos x$

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Derivatives of Sinusoidal Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

Derivatives of composite sinusoidal Functions

If $y = \sin f(x)$, then $\frac{dy}{dx} = \cos f(x) \times f'(x)$

In Leibniz notation, $\frac{d}{dx}(\sin f(x)) = \frac{d(\sin f(x))}{d(f(x))} \times \frac{d(f(x))}{dx}$

$= \cos f(x) \times \frac{d(f(x))}{dx}$

If $y = \cos f(x)$, then $\frac{dy}{dx} = -\sin f(x) \times f'(x)$

In Leibniz notation, $\frac{d}{dx}(\cos f(x)) = \frac{d(\cos f(x))}{d(f(x))} \times \frac{d(f(x))}{dx}$

$= -\sin f(x) \times \frac{d(f(x))}{dx}$

- When you are differentiating a function that involves sinusoidal functions, use the rules given above, along with the sum, difference, product, quotient and chain rules as required.

Practice

1. a) $y' = 2\cos 2x$
   b) $y' = -2\sin 3x\,(3)$
   c) $y' = \cos(x^3 - 2x + 4)(3x^2 - 2)$
   d) $y' = -2(\sin(-4x))(-4)$
   e) $y' = 3\cos 3x + 4\sin 4x$
   f) $y' = 2^x \ln 2 - 2\cos x + 2\sin x$
   g) $y' = e^x \cos(e^x)$
   h) $y' = 3\cos(3x + 2\pi)(3)$
   i) $y' = 2x + \sin x + \cos\frac{\pi}{4}$
   j) $y' = \cos(x^{-1})(-x^{-2})$