5.5 The Derivative of $y = \tan x$

Derivatives of Composite Functions involving $y = \tan x$

If $y = \tan f(x)$, then $\frac{dy}{dx} = \sec^2 f(x) \times f'(x)$

In Leibniz notation, $\frac{d}{dx}(\tan f(x)) = \frac{d(\tan f(x))}{d(f(x))} \times \frac{df(x)}{dx}$

$= \sec^2(f(x)) \times \frac{d(f(x))}{dx}$

- The derivatives of functions involving the tangent function are found as follows →

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d}{dx}(\tan(f(x))) = \sec^2 f(x) \times f'(x)$$

- Trigonometric identities can be used to write one expression as an equivalent expression and then differentiate. In some cases, the new function will be easier to work with

~~Q.13~~
ex $y = \sin(e^{2^x})$

$y' = \sin(e^{2^x})(e^{2^x})(2^x)\ln 2$

$y = \sin(\cos(\sin e^{2x}))$

$y' = \cos(\cos(\sin(e^{2x})) \cdot (-\sin(\sin e^{2x})) \cdot \cos e^{2x} \cdot e^{2x} \cdot 2$

ex $y = \frac{\sin x}{\cos x}$ $y' = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

$y = \tan\square$ $y' = \sec^2\square \cdot \square'$

ex. $y = \tan(x^3 + x^2 - x + 1)$ $y' = \sec^2(x^3 + x^2 - x + 1) \cdot 3x^2 + 2x - 1$

$y = \tan(1/x) \cdot x^2$ $y' = 2x\tan(1/x) \cdot x^2\sec^2(1/x)(-1/x^2)$

$y = \tan^3(\sqrt{x})$ $y' = 3(\tan^2\sqrt{x}) \cdot \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$y = 3\sqrt{x} \cdot \tan e^{x^2}$ $\quad y' = \frac{1}{3} x^{-2/3} \tan e^{x^2} + \sqrt[3]{x} \sec^2(e^{x^2})(e^{x^2}) \cdot 2x$

$y = \frac{\tan(2^x)}{e^{2x}}$ $\quad y' = \frac{\sec^2(2^x) \cdot 2^x \cdot \ln 2 \cdot e^{2x} - 2e^{2x}\tan(2^x)}{e^{4x}}$

$= \frac{\cancel{e^{2x}}(\sec^2(2^x)(2^x) \cdot \ln 2 - 2\tan(2^x))}{\cancel{e^{2x}} \cdot e^{2x}}$